Index

adapted process, 108, 135, 183
ADI, *see* alternating direction implicit
alternating direction implicit, 104, 115
analytic solution, 30, 45, 85, 86
antithetic variates, 90
Apple, 159
arbitrage, 2, 3, 8, 19–21, 34, 53, 60, 61, 86, 109, 110, 123, 137, 164, 170, 180
arbitrage-free price, 25, 35
at-the-money, 28, 51

barrier
  continuous, 93, 101, 116, 125, 127
  discretely monitored, 49, 93, 101, 149
  knock-in, 45, 59, 101, 107, 130
  knock-out, 45, 59, 107, 130
  lower, 49
  single, 49
  upper, 49
  window, 122
barrier option, *see* option, barrier
bisection search, 83
Black-Scholes, 4, 26, 40, 42, 43, 51, 52, 54, 59, 73, 118, 121, 136, 160, 177
Black-Scholes equation, 29–30, 36, 37, 59, 61
Black-Scholes formula, 26–29, 31, 34, 44, 51, 53, 61, 86, 112, 113, 129, 142, 148, 150, 170
Black-Scholes process, 48, 78
bond, 61
boundary condition, 30, 36, 37, 59, 68, 69, 74
  Feller, 71–73
  zero gamma, 96
  zero price, 95
Box-Muller transform, 88
Breeden-Litzenberger, 56, 116, 137, 170
broker, 51, 109, 157
Brownian motion, 9–12, 32, 45, 65, 77, 87, 92, 108, 117, 119, 154, 163, 183
  correlated, 17, 161
bump function, 58

calendar spread, 110, 111
calibration, 63, 64, 71, 124–125
call, 3
call spread, 56, 91
callable trade, 93, 101, 106
cash settlement, 2, 157
cell averaging, 101
central bank, 5
central difference, 95, 103, 104
CEV, *see* constant elasticity of variance
Chase Manhattan, 131
Cholesky decomposition, 90
CIR, *see* Cox-Ingersoll-Ross model
close out, 2, 147
CMS, *see* constant maturity swap
cockroach, 141
commodities, 2, 8
constant elasticity of variance, 74
constant maturity swap, 158
continuous barrier option, *see* option, barrier
copula, 154–155, 170
correlation, 17, 54, 64, 89
  asset-asset, 153, 154
  implied, 156, 157, 160–162, 175
  instantaneous, 153, 166
  local-stochastic, 166
  spot-volatility, 65, 66, 73, 84, 113, 116, 117, 122, 127, 130, 142, 165
correlation matrix, 164
correlation product, 153
correlation smile, 155–162, 173–175
cost of carry, 8
Cox-Ingersoll-Ross process, 66, 71, 72
Craig-Sneyd scheme, 105
Crank-Nicolson scheme, 97, 99, 105
credit rating, 5
cross asset, 160

cross smile, 160, 162–164, 170, 173
cross volatility surface, *see* cross smile
cumulative normal function, 28, 46
    bivariate, 155, 168
    inverse, 86

delta, 38, 85, 138, 139, 141
    Black-Scholes, 52, 112, 137, 161
    forward, 52, 137
delta function, *see* Dirac delta function
delta hedging, *see* hedging, delta
derivative, 1, 3
derivatives trading, 40
digital payout, *see* option, digital
Dirac delta function, 56, 69, 70, 110, 115, 172
discount factor, 27, 31, 33, 55, 56, 67, 86
discounted expectation, 27, 34
dividend, 8, 19, 20, 61
domestic currency, 6
drift, 12, 16, 21, 47, 61, 121, 134, 161, 163, 183, 184
    terminal, 16
Dupire local volatility model, *see* local volatility model
dynamic trading strategy, 36

efficient market, 2
eigenvalue, 98
energy, 2
equities, 2, 8, 51, 54, 141, 153
equivalent probability measure, *see* measure, equivalent
euro-dollar, 51, 53, 153
European payout, *see* option, European
exchange, 2
expectation, 180
expiry date, 3, 51, 82, 83
expiry time, *see* expiry date
explanatory variables, 94
explicit scheme, 96
exponential Ornstein-Uhlenbeck model, *see* Ornstein-Uhlenbeck process

factorisation method, 104
federal funds rate, 5
Feller boundary condition, *see* boundary condition, Feller
filtration, 15, 58, 108, 135, 147, 183
financial engineer, 183
financial mathematician, 183
finite difference method, 86, 94, 104, 106, 115, 116, 145
first generation exotic option, *see* option, first generation exotic
fixed income, 6
floating smile, 112, 113
Fokker-Plank equation, *see* forward Kolmogorov equation
foreign exchange, 2, 8, 33, 51, 53, 63, 82, 112, 125, 140, 141, 153, 157, 160
forward contract, 1, 6–8, 19, 27, 40, 52, 53, 57, 107
forward induction, 114, 116, 124, 125, 131, 177
forward Kolmogorov equation, 58–60, 62, 71, 84, 115
forward level, 1, 8, 27, 28, 33, 35, 43, 44, 121, 140, 146, 147
forward start, *see* option, forward starting
forward volatility agreement, 146–151
    asset, 147
    cash, 147
Fourier transform, 69, 70
FTSE, 54, 63
fundamental theorem of on-smile pricing, 108–110, 114, 146
FVA, *see* forward volatility agreement
FX, *see* foreign exchange

gamma, 38–40, 85, 139
Gauss-Hermite integration, 122
Gaussian, *see* normal variable
Girsanov's theorem, 22–24, 34, 45, 47, 179–184
going short, *see* short position
gold, 8, 19, 107, 158, 160, 165, 172, 173
government bond, 20
Greeks, 38–39, 41, 85, 106, 123, 138, 177
Gyöngy's theorem, *see* fundamental theorem of on-smile pricing

heat kernel, 75
hedge fund, 40, 140
hedging, 1, 85, 123, 141, 178
    delta, 37–38, 40, 41, 63, 133, 137, 177
    dynamic, 35–37, 140
hedging portfolio, 36
hedging strategy, 38
Heston model, 65–73, 75, 78, 83, 84, 88, 106

IBM, 29, 157, 159

implicit scheme, 97, 103, 106, 115, 131
implied volatility, *see* volatility, implied
implied volatility dynamic, 111, 112
implied volatility smile, *see* smile
implied volatility surface
    see volatility surface, 51
in-the-money, 29, 35, 51, 141
inflation, 20
insurance company, 35
integral
    Lebesgue, 181, 182
    Riemann, 181
integration by parts, 149
interest rate, 5–6, 19, 51, 61, 121, 126, 140
    continuously compounding, 5, 19
    stochastic, 87
interfacing, 102, 145
investment bank, 35, 40, 42, 122, 133, 140, 160, 165
Ito calculus, 12
Ito process, 13, 34, 179
Ito's lemma, 13–14, 16–17, 36, 58, 67, 73, 74, 109, 115, 118, 135, 184
    product rule, 14, 26, 184

Jacobian, 88
joint probability distribution, *see* probability distribution, joint, 93
JP Morgan, 131
jumps, 110, 135, 140

knock-in barrier, *see* barrier, knock-in
knock-out barrier, *see* barrier, knock-out

lambda-SABR model, 75, 84, 114, 118, 124, 126, 127, 165
Laplace transform, 71, 72
law of large numbers, 87, 88
Lehman bankruptcy, 63
Lévy's theorem, 184
linear congruential generator, 87
liquid asset, 3
local correlation model, 162–164, 170, 172
local stochastic correlation model, 165–166, 172
local stochastic volatility model, 54, 84, 113–116, 121, 122, 125, 127, 130, 131, 135, 143–145, 149, 165, 177
local volatility correction, 114–118, 121, 122, 124, 125, 130, 131
local volatility model, 54, 58–62, 82, 84, 86, 94, 107, 109, 111, 112, 118, 123–127, 130, 149, 172
    multi-asset, 153, 155
log contract, 136, 152
log return, 134, 137, 139
log-normal, 14–15, 26, 27, 43, 44, 47, 67, 75, 87, 94, 114, 119, 120
long expiry date, 84, 140
Longstaff-Schwarz, 94
low discrepancy random numbers, *see* random numbers, low discrepancy
LSV, *see* local stochastic volatility model
LVCC, *see* local volatility, multi-asset

market data bump, 85
market maker, 40, 109, 160
Markov chain, 79
Markov property, 15
Martingale, 24–26, 30, 34, 61, 67, 135, 136, 184
    continuous, 25
Martingale pricing, 19
Martingale pricing equation, 25, 27, 32, 86, 105, 169
matching delta, 173, 174
mean reversion, 63, 64, 66, 72, 75–77, 84, 117, 118, 121, 123–125, 127, 151, 152
mean reversion level, 64
measure, 21, 26, 34, 180
    equivalent, 22, 181, 183
    real world, 183
    risk-neutral, 64, 66, 148, 166
measure change, 21–24, 119, 161, 166, 169, 179–184
measure theory, 179
Mersenne twister algorithm, 87
method of characteristics, 72
method of planes, 102, 106, 144, 150
mixing parameter, 114, 124–131, 144, 145, 152
mixture model, 77–79, 122, 123, 126, 127, 131, 145, 166
model uncertainty, 178
moment matching, 44, 50
money market account, *see* rolling money market account
Monte Carlo, 30, 86–94, 105, 106, 144, 154, 164, 167
    error, 90
    Greeks, 91

multiple dimensions, 88
standard deviation, 88
variance reduction, 90
multiple assets, 16, 153

Newton-Raphson method, 86
no arbitrage principle, *see* arbitrage
normal process, 76, 119, 127
normal variable, 9, 15, 87, 88, 122, 154, 155
notional, 2, 35
numeraire, 21, 25, 26, 29, 31, 33, 34, 48, 49, 61, 68, 86, 161, 163, 166, 169, 172
numeraire symmetry, 172–173
numerical integration, 65, 86, 92, 122
numerical stability, 78, 83, 85

ODE, *see* ordinary differential equation
off-smile, 126
oil, 2
OIS, *see* over night indexed swap
one-sided differencing, 96
optimisation problem, 82
option
American, 93
Asian, 43–45, 50, 102, 106, 144
at-the-money, 28, 40, 54, 63, 129, 142, 147
barrier, 45–50, 59, 114, 116, 118, 119, 121, 122, 124, 125, 130, 131, 142, 151
basket, 153, 157, 171, 173–175
best-of, 34, 158, 167–169, 171
call, 19, 26, 31, 38, 48, 54, 60, 63, 68, 109, 153
composite, 159, 162, 163, 172, 173, 175
digital, 31, 43, 55–57, 68, 91, 116, 128, 129, 149
dual asset digital, 171
European, 42–43, 50, 55, 57, 61, 86, 109, 122, 131, 154, 167
exercise, 3
exotic, 2, 4, 59, 64, 107, 114, 123, 124, 126, 130, 131
first generation exotic, 116, 122, 131, 143, 144
forward starting, 123, 147
geometric basket, 159
in-the-money, 40, 54
no-touch, 130, 149, 165
one-touch, 124–128, 130
out-of-the-money, 40, 54, 63
put, 54, 55, 63
quanto, 157
short dated, 39, 52
spread, 158
vanilla, 3, 29, 38, 43, 54–57, 59, 61, 64, 65, 67, 82, 86, 107, 116, 133, 136, 140
worst-of, 158, 171
ordinary differential equation, 69
Ornstein-Uhlenbeck process, 76–77, 84, 125
OU process, *see* Ornstein-Uhlenbeck process
out-of-the-money, 29, 51, 141
overnight, 39
overnight indexed swap, 5

parallel shift, 151
parameter averaging, 84
partial differential equation, 29, 30, 34, 58, 59, 61, 64, 67–69, 81, 115, 124, 126, 131, 154
multiple dimensions, 103–105
numerical solution, 30, 83, 86, 94–106, 144, 166, 177
parabolic, 95
stability, 96–100
payout, 3
PDE, *see* partial differential equation
PDE approach, *see* partial differential equation, numerical solution; *see also* finite difference method
PDE solver, 101–103, 106, 115, 116, 131, 150, 166, 167
PDF, *see* probability density function
percentage notional, 29
physical settlement, 157, 159
positive definite matrix, 164
pounds sterling, 20
precious metals, 2
predictor-corrector method, 104
present value, 27, 121
probability, 181
risk-neutral, 68
probability density function, 55, 58, 59, 71, 72, 167
probability distribution, 58, 59, 108
cumulative, 55
implied, 55, 86

joint, 45, 148, 154, 169–172
marginal, 148, 154, 155
probability measure, *see* measure
probability space, 180, 182, 183
product rule, *see* Ito's lemma, product rule
pseudo-random numbers, *see* random numbers, pseudo- random
put, 3
put-call parity, 53, 147, 148, 159, 171
PV, *see* present value

quant, *see* quantitative analyst
quantitative analyst, 2, 27, 33, 150, 183
quasi-random numbers, *see* random numbers, low discrepancy

Radon-Nikodým derivative, 22, 47, 48, 119, 182, 183
random numbers
low discrepancy, 92
pseudo-random, 87, 92
random variable, 180
Rannacher stepping, 100
real world probability measure, *see* measure, real world
real world process, 19, 21, 22, 26, 35, 37
realised variance, 133, 136, 141, 145
reflection principle, 45–48, 50, 119
regime switching model, 79–81, 145, 166
regularisation, 164
replicating portfolio, 7
replication, 7, 35, 40, 56, 57, 86, 128, 140, 141, 149, 152, 177
static, 178
Reuters, 133
rho, 38
risk, 85
risk free rate, 5, 20
risk management, 122
risk manager, 85, 176, 178
risk reversal, 82
risk-neutral expectation, 136, 147
risk-neutral measure, 27, *see* measure, risk-neutral
risk-neutral probability, 52, 56, 60, 61, 137
risk-neutral process, 26, 31, 42
rolling money market account, 20, 25–27, 29, 31, 42, 67

SABR formula, 74, 75, 82
SABR model, 73–75, 78, 82–84, 113, 118, 125
sample space, 179–181
SDE, *see* stochastic differential equation
seed, 91
self-financing portfolio, 36, 37
short expiry date, 84
short position, 35
shorting, *see* short position
sigma algebra, 179–182
silver, 165, 172, 173
skew, 51, 54, 63, 64, 82, 83, 125, 129, 130, 138, 142
smile, 45, 51–55, 59, 61, 63–65, 74, 77, 78, 83, 84, 107, 111, 112, 124, 126, 129, 130, 133, 137, 138, 142–144, 148, 149, 151, 160, 170, 178
convexity, 63–66, 74, 77, 78, 82–84, 118, 124, 125, 138
interpolation, 52
Sobol sequence, 92
speculation, 2
spot, 2
spot dynamic, 54, 59
spot price, *see* spot
spread, 40
standard deviation, 95
step function, 31, 55
sterling-dollar, 153, 157
sticky delta, 112–113, 124, 125, 138
sticky strike, 111–112, 124, 125, 138
stochastic differential equation, 12, 58, 62, 65, 73, 74, 118, 163, 173
stochastic integral, 9, 22
stochastic process, 9
stochastic variance, 71
stochastic volatility model, 54, 63–65, 73, 78, 83, 107, 111, 113, 114, 116, 118, 122–124, 127, 130, 142, 143, 149, 151
calibration, 82–84
straddle, 146, 147
strangle, 82
strike, 2, 35, 51, 55, 60, 82, 111
at-the-money, 51
supply and demand, 53–55
SV model, *see* stochastic volatility model
swap rate, 5

Taylor expansion, 29
term structure, 6, 16, 28, 42, 64, 118, 120–123, 126
terminal volatility, 118
theoretical value, 126, 127

theta, 38, 40
theta scheme, 97
Thomas algorithm, 97, 104
time decay, 39
time value of money, 111
tradeable asset, 20, 25, 26, 31
trader, 27, 37, 42, 85, 91, 124–126, 130, 141, 149, 151, 174, 176
trading strategy, 35
triangle rule, 160–163, 165, 166, 173
TV, *see* theoretical value

unconditional stability, 100, 105
United States dollar, 2, 20

vanilla, *see* option, vanilla
vanilla market, 59, 107–109, 113, 124
vanilla payout, *see* option, vanilla
variance reduction, *see* Monte Carlo, variance reduction
variance swap, 57, 133–141, 143, 151
vega, 38–40, 85, 118, 121, 129, 141, 150, 151, 178
vol-of-vol, *see* volatility of volatility
volatility, 4, 12, 16, 21, 37, 40, 47, 63, 133
  at-the-money, 51, 83, 126, 129, 138, 143, 147
  fair, 134, 141, 143
  implied, 45, 51, 53, 54, 56, 59, 61, 63, 74, 111, 114, 130, 133, 137, 142, 146, 147, 150, 160
  instantaneous, 63, 83, 84, 113, 114, 117, 120, 146
  interpolation, 106, 138
  local, 58–61, 108, 110, 112, 114, 117, 130, 146, 163
  realised, 64, 65, 123, 133, 147
  stochastic, 63, 124
  terminal, 16, 121, 122, 131
volatility frown, 55
volatility of variance, 66, 72
volatility of volatility, 63, 64, 73, 74, 77, 83, 117, 118, 121–125, 131, 151
volatility process, 64
volatility smile, *see* smile
volatility surface, 51, 52, 54, 85, 108, 110, 116, 126, 130, 148, 165, 178
  arbitrage, 109
volatility swap, 141–146, 151
  instantaneous, 146
volatility trading, 40

wavelets, 106
wheat, 1, 8
winding number, 70
wingyness, 140, 143, 144

yield, 6, 19, 29
yield curve, 6, 8, 85

zero coupon bond, 20, 67, 68