

# Contents

List of symbols	xi
Acknowledgements	xiii
Preface	xiv
<b>1 Introduction to Derivatives</b>	<b>1</b>
1.1 Hedging with Forward Contracts	1
1.2 Speculation with Forward Contracts	2
1.3 Arbitrage	2
1.4 Vanilla Options	3
1.5 Interest Rates	5
1.6 Valuing a Forward Contract	6
1.7 Key Points	8
1.8 Further Reading	8
<b>2 Stochastic Calculus</b>	<b>9</b>
2.1 Brownian Motion	9
2.2 Stochastic Model for Stock Price Evolution	12
2.3 Ito's Lemma	13
2.4 The Product Rule	14
2.5 Log Normal Stock Price Evolution	14
2.6 The Markov Property	15
2.7 Term Structure	16
2.8 Ito's Lemma in More than One Dimension	16
2.9 Key Points	18
2.10 Further Reading	18
<b>3 Martingale Pricing</b>	<b>19</b>
3.1 Setting the Scene	19
3.2 Tradeable Assets	20
3.3 Zero Coupon Bond	20
3.4 Rolling Money Market Account	20
3.5 Choosing a Numeraire	21
3.6 Changing the Measure	21
3.7 Girsanov's Theorem	22
3.8 Martingales	24
3.9 Continuous Martingales	25

3.10	Black-Scholes Formula for a Call Option	26
3.11	At-The-Money Options	28
3.12	The Black-Scholes Equation	29
3.13	An Elegant Derivation of the Black-Scholes Formula	31
3.14	Key Points	34
3.15	Further Reading	34
4	Dynamic Hedging and Replication	35
4.1	Dynamic Hedging in the Absence of Interest Rates	35
4.2	Dynamic Hedging With Interest Rates	37
4.3	Delta Hedging	37
4.4	The Greeks	38
4.5	Gamma, Vega and Time Decay	39
4.6	Vega and Volatility Trading	40
4.7	Key Points	40
4.8	Further Reading	41
5	Exotic Options in Black-Scholes	42
5.1	European Options	42
5.2	Asian Options	43
5.3	Continuous Barrier Options	45
5.3.1	The Reflection Principle	45
5.3.2	The Reflection Principle with Log-Normal Dynamic	47
5.3.3	Valuing Barrier Options in Black-Scholes	48
5.3.4	Discretely Monitored Barrier Options	49
5.4	Key Points	50
5.5	Further Reading	50
6	Smile Models	51
6.1	The Volatility Smile	51
6.2	Smile Implied Probability Distribution	55
6.3	The Forward Kolmogorov Equation	58
6.4	Local Volatility	59
6.5	Key Points	61
6.6	Further Reading	62
7	Stochastic Volatility	63
7.1	Properties of Stochastic Volatility Models	64
7.2	The Heston Model	65
7.2.1	What Makes the Heston Model Special	65
7.2.2	Solving for Vanilla Prices	68
7.2.3	The Feller Boundary Condition	71
7.3	The SABR Model	73
7.4	The Ornstein-Uhlenbeck Process	76
7.5	Mixture Models	77
7.6	Regime Switching Model	79
7.7	Calibrating Stochastic Volatility Models	82
7.8	Key Points	84

7.9	Further Reading	84
<b>8</b>	<b>Numerical Techniques</b>	<b>85</b>
8.1	Monte Carlo	86
8.1.1	Monte Carlo in One Dimension	86
8.1.2	Monte Carlo in More than One Dimension	88
8.1.3	Variance Reduction in Monte Carlo	90
8.1.4	Limitations of Monte Carlo	92
8.2	The PDE Approach	94
8.2.1	Stable and Unstable Schemes	96
8.2.2	Choice of Scheme	100
8.2.3	Other Ways of Improving Accuracy	100
8.2.4	More Complex Contracts in PDE	101
8.2.5	Solving Higher Dimension PDEs	103
8.3	Key Points	105
8.4	Further Reading	106
<b>9</b>	<b>Local Stochastic Volatility</b>	<b>107</b>
9.1	The Fundamental Theorem of On-smile Pricing	108
9.2	Arbitrage in Implied Volatility Surfaces	109
9.3	Two Extremes of Smile Dynamic	111
9.3.1	Sticky Strike Dynamics	111
9.3.2	Sticky Delta Dynamics	112
9.4	Local Stochastic Volatility	113
9.5	Simplifying Models	116
9.5.1	Spot-Volatility Correlation	117
9.5.2	Term Structure Vega for a Barrier Option	118
9.5.3	Simplifying Stochastic Volatility Parameters	121
9.5.4	Risk Managing with Local Stochastic Volatility Models	122
9.6	Practical Calibration	124
9.7	Impact of mixing on contract values	125
9.8	Key Points	130
9.9	Further Reading	131
<b>10</b>	<b>Volatility Products</b>	<b>133</b>
10.1	Overview	133
10.2	Variance Swaps	133
10.2.1	The Variance Swap Contract	133
10.2.2	Idealised Variance Swap Trade	134
10.2.3	Valuing the Idealised Trade	135
10.2.4	Beauty in Variance Swaps	136
10.2.5	Delta and Gamma of a Variance Swap	138
10.2.6	Practical Considerations	140
10.3	Volatility Swaps	141
10.3.1	Volatility Swap in Stochastic Volatility Models and LSV	142
10.3.2	Volatility Swap Versus Variance Swap	143
10.3.3	Valuing a Volatility Swap	144
10.3.4	Stochastic Versus Local Volatility	145

10.4 Forward Volatility Agreements	146
10.4.1 Practicalities	150
10.5 Key Points	151
10.6 Further Reading	152
<b>11 Multi-Asset</b>	<b>153</b>
11.1 Overview	153
11.2 Local Volatility with Constant Correlation	153
11.3 Copulas	154
11.4 Correlation Smile	156
11.5 Marking Correlation Smile	156
11.5.1 Common Correlation Products	156
11.5.2 The Triangle Rule	160
11.6 Modelling	162
11.6.1 Local Correlation	162
11.6.2 Practicalities	163
11.6.3 Local stochastic correlation	165
11.7 Valuing European Contracts	167
11.7.1 Special Properties of Best-of Options	167
11.7.2 Valuing a Best-of Option in Black-Scholes	168
11.7.3 Construction of a Joint PDF	169
11.7.4 Using the Density Function for Pricing	171
11.8 Numeraire Symmetry	172
11.9 Baskets as Correlation Instruments	173
11.10 Summary	175
11.11 Key Points	176
11.12 Further Reading	176
Afterword	177
Appendix: Measure Theory and Girsanov's Theorem	179
References	185
Further Reading	191
Index	194